

12.2 Vector Intro ([2D Desmos](#) | [3D Desmos](#))

Definition & Notation

A vector represents magnitude and direction. We depict it with an arrow:

- magnitude = "arrow length"
- initial point = "tail"
- terminal point = "head"

Components record the changes in x, y, and z from tail to head.

$$\langle \Delta x, \Delta y, \Delta z \rangle$$

Entry Task (similar to HW #1–2)

Consider the 2D vector $\langle 2, -1 \rangle$

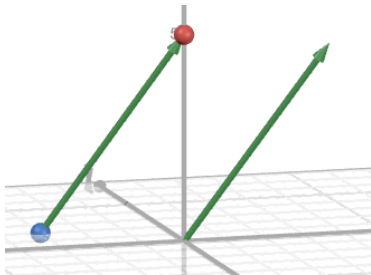
1. Draw the same vector with its tail at (5,3).
2. Draw the same vector with its tail at (0,0).

A vector can move. Its magnitude and direction determine the vector - not where the arrow is drawn.

Components & Basic Facts (HW #2, #5–6)**Magnitude**

The vector from $A(a_1, a_2, a_3)$ to $B(b_1, b_2, b_3)$ is

$$\overrightarrow{AB} = B - A = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle$$



You do:

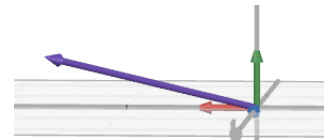
Find the vector from $A(2, 3, 0)$ to $B(0, 0, 5)$.

$$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

Find the magnitude of $\overrightarrow{AB}$

Standard basis vectors

$$\mathbf{i} = \langle 1, 0, 0 \rangle \quad \mathbf{j} = \langle 0, 1, 0 \rangle \quad \mathbf{k} = \langle 0, 0, 1 \rangle$$



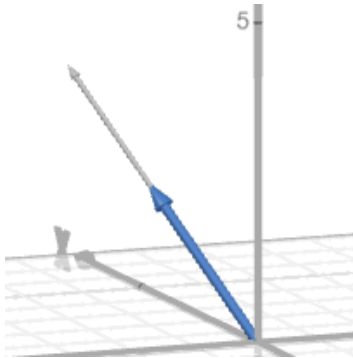
Write $\overrightarrow{AB}$ as a linear combination of the standard basis vectors

Scaling & Unit Vectors (HW #7–9)

If c is a scalar, then

$$c\langle v_1, v_2, v_3 \rangle = \langle cv_1, cv_2, cv_3 \rangle$$

Scaling changes length; a negative scalar also reverses direction.



Unit vectors

A unit vector has length 1.

$$\text{unit vector in direction of } \mathbf{v} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

You do:

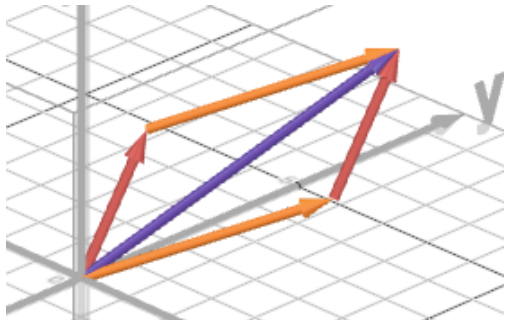
- (a) Give a unit vector in the direction of $\langle 3, 2, 4 \rangle$.
- (b) Give a vector opposite $\langle 3, 2, 4 \rangle$ and three times as long.

Vector Addition & Subtraction (HW #3–6)**You do - (HW #1)**

$$\mathbf{v} + \mathbf{w} = \langle v_1 + w_1, v_2 + w_2, v_3 + w_3 \rangle$$

Addition visually:

move vectors head-to-tail or use the parallelogram rule.



Subtraction visually:

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$$



Using the vectors above, sketch:

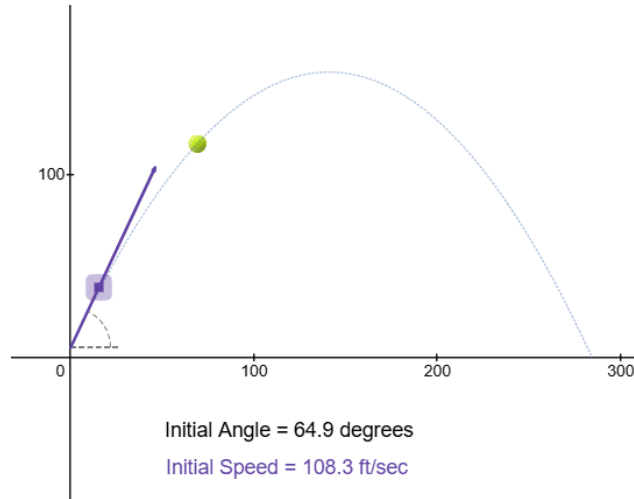
- (a) $\mathbf{u} + \mathbf{w}$
- (b) $\mathbf{u} + \mathbf{v} + \mathbf{w}$
- (c) $\mathbf{u} - \mathbf{v}$

Compare your drawings with someone nearby.

Magnitude + Direction in 2D (HW #10–11)

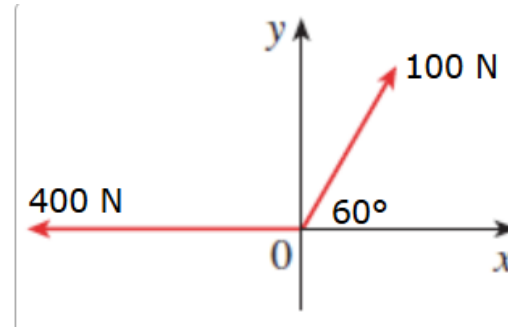
If a vector has magnitude a and makes angle θ with the positive x-axis:

$$\mathbf{v} = \langle a \cos(\theta), a \sin(\theta) \rangle$$



Resultant Force (HW #11) [2D Desmos](#)

"Resultant force" means vector sum.



Vectors from Slopes (HW #12–13)

If a line has slope m , then a parallel direction vector is

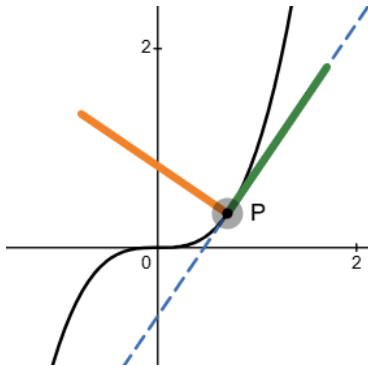
$$\langle 1, m \rangle$$

A perpendicular direction vector is

$$\langle -m, 1 \rangle$$

You do

Find both unit vectors that are parallel to the tangent line to $y = x^2$ at $(4, 16)$. Then find the unit vectors perpendicular to the tangent line.

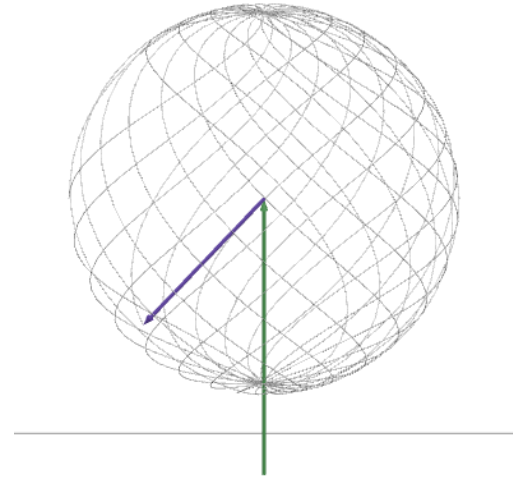


Thinking in terms of vectors (HW #13)

If $\mathbf{r}_0 = \langle 0, 0, 3 \rangle$ and $\mathbf{r} = \langle x, y, z \rangle$

Then describe the collection of all x, y, z values that would make the following true...

$$|\mathbf{r} - \mathbf{r}_0| = 2$$



Summary of Key Facts from 12.2

Fundamental Concepts

Think of vectors as $\langle \Delta x, \Delta y, \Delta z \rangle$

$$\overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle$$

$$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$\mathbf{i} = \langle 1, 0, 0 \rangle \quad \mathbf{j} = \langle 0, 1, 0 \rangle \quad \mathbf{k} = \langle 0, 0, 1 \rangle$$

Scaling and Addition Ideas

$$c\langle v_1, v_2, v_3 \rangle = \langle cv_1, cv_2, cv_3 \rangle$$

$$\text{unit vector in direction of } \mathbf{v} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

$$\mathbf{v} + \mathbf{w} = \langle v_1 + w_1, v_2 + w_2, v_3 + w_3 \rangle$$

2D vector facts

$$\mathbf{v} = \langle a \cos(\theta), a \sin(\theta) \rangle$$

Vector parallel to a line with slope m : $\langle 1, m \rangle$